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Fatigue tests for automotive design: optimization of the test protocol and improvement of the fatigue strength parameters estimation

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Abstract

Despite of continuous improvement of numerical computations, experimental fatigue tests are still of prior concern for high safety level parts, in particular for the automotive industry. Destructive fatigue tests are required in order to prove to authorities an effective fatigue design by highlighting the weakest link and its failure mode, and possibly taking account for the manufacturing process influence. Actually, experimental tests need for expensive means (i.e. rigs and specimens) and fatigue phenomenon is expected to take long time to get a crack issue. Thus, cost and delay time reduction always calls for more effective methods.

Within this framework, we address here some recent improvements dealing with, respectively: a) the optimization of the fatigue test protocol, i.e. how the load time history is applied to the specimen in order to minimize the overall fatigue test duration; b) the reliability of the fatigue strength estimation, i.e. how the experimental data are considered in a reliable statistical model. The first point let us focus on a so-called Locati fatigue test protocol, whereas the second point let us make use of Maximum Likelihood and Bayesian techniques.

These improvements are currently and successfully applied to chassis system parts at PSA Peugeot Citroën.

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Keywords: automotive chassis parts; fatigue; reliability; test protocol; maximul likelihood estimation; Bayesian inference.

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Nomenclature

A	material constant of Basquin model
L	likelihood function
N	load cycle number
S	load cycle amplitude
T	scatter parameter
X	random variable, e.g. a fatigue test results
b	Basquin slope
g	a priori / a posteriori function
n	number of specimens
q	variation coefficient or relative dispersion
γ	confidence level
μ	mean value
σ	standard deviation or square root of the variance
ϑ	estimated parameter

1. Introduction

In this paper we deal with fatigue applied to automotive passenger vehicles. In particular, we focus on the chassis system, i.e. front and rear axles as shown in figure 1. Fatigue is not the only design concern, nevertheless it is relevant indeed: in terms of orders of magnitude, considering a car life target of about 10^5 km, the customer usage analysis leads us to count up to 10^6 braking and cornering events, thus 10^6 cyclic load occurrences, calling for high cycle fatigue. For the sake of simplicity, within this framework we concentrate our attention on usual driving situations only, coming from everyday life customer usage, thus neglecting any overload related to special driving conditions or misuse.

Fig. 1. Automotive chassis system: front and rear axles of a passenger car, Peugeot 508 (left) and Peugeot RCZ (right).

Chassis system is composed of safety parts, whose reliability with respect to fatigue is expected to be proven. Despite of the continuous and increasing performance and relevancy of numerical computations, experimental evidence of chassis parts fatigue strength is still mandatory: at most, numerical computations help us to identify and select the most relevant fatigue test to be provided.

Looking at the overall vehicle, the common testing strategy calls for a final test, on proving ground or testing rig, applied at the very last stage of the design process as a final check. Usually, one single car specimen composed of thousands of parts faces to a lifetime target to be experienced without any failure by all the car parts. Please note that even if such a test is successful, it is not expected to provide enough information to assess high level safety requirements.

Prior to success such a global final test and in order to assess reliable fatigue strength estimation, each single chassis part is expected to experience preliminary fatigue tests. At this stage, several specimens are supposed to be available, leading to a reliable statistical approach. Usually, these fatigue tests are not censored, but applied up to failure: this because of the need to know the failure mode (i.e. the weakest link) and the very upper threshold of the actual design. From a purely statistic point of view, making the failure information available leads to extract the richest data from the test.

Within this framework, the key feature is to set an effective compromise between the number of specimens, i.e. the test duration and cost, and the statistical estimation relevance, which is a long-lasting challenge indeed.

This paper is organized as follows: at first, we introduce the basic ingredients used to describe high cycle fatigue, i.e. fatigue strength and lifetime models in the Wöhler plane (§2). Then, a brief review of the three main ingredients of a fatigue test campaign is presented (§3), i.e. the test specimen number, the test protocol and the test estimation method. Particular attention is paid to the choice of the best test protocol (§3.2), the application of the Maximum Likelihood Estimation (§3.3). At last, discussions and conclusions end the paper, focusing on the application of the Bayesian inference technique (§4).

2. Fatigue strength and lifetime models in the Wöhler plane

We consider high cycle fatigue phenomenon [1,2], i.e. applying cyclic loads below the elastic to plastic behavior threshold of the materials used for chassis system parts. Moreover, we focus on almost constant amplitude tests, i.e. the so-called Wöhler framework. The most basic model used to express a relationship between the load cycle amplitude, S with respect to the life cycle number, N is that of Basquin [3], as expressed in the following equation:

$$NS^b = A \tag{1}$$

where the Basquin slope, b and A are material constants. At least for metallic materials, this model is supposed to hold from 10^4 up to 10^6 cycles and leads to a linear relation in the logS-logN plane, see figure 2.

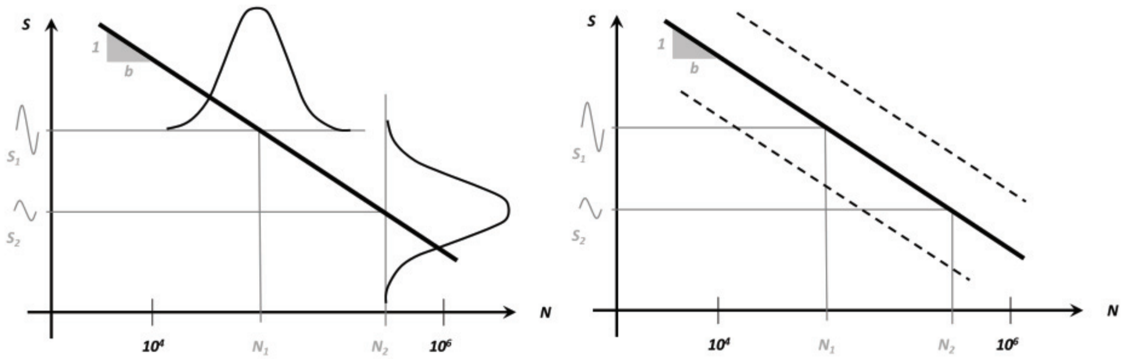

Fig. 2. Basquin model in the logS-logN plane: stress and lifetime models (left); iso-survival probability lines (right).

This model is applied to deal with the mean fatigue strength at each load level. Fatigue is a dispersive phenomenon, thus it is of prior concern to complete the previous linear model with a statistic distribution model over it. There exist two points of view, as displayed in figure 2, left side: the first one comes from experimental test practice, where S is fixed prior to get a distribution for the cycle number N; the second comes from customer to design requirements, where a product lifetime is expected and a load level S distribution is thus considered for design. Moreover, the latter point of view is widely applied in the framework of the stress-strength interference method [4,5]: the strength distribution is considered with respect to the stress one along the S axis in order to assess the design reliability, which is of prior concern for safety parts.

At least since the early work of Bastenaire [6], but also stated by fatigue test normalization for material specimen [7,8], it is usually accepted that for a fixed S, N follows a log-normal law. Besides, in the stress-strength interference method, for a fixed N, S usually follows a normal or Gaussian law [4,5]. In some applications, a Weibull law is also used as an effective model [9]. The common experience learns that looking at the population kernel, i.e. in order to describe the central portion of the population, the choice of the particular density function model is far from a sensible feature [10]. Conversely, when dealing with failure probability estimation and especially looking at very low values, i.e. the induced density function tails, the choice of the particular density function model is significant indeed [9].

Anyway, all of these statistical models are described by two parameters, i.e. a mean value, μ and a scatter, expressed by the standard deviation, σ which is the square root of the variance, σ^2. The scatter may also be measured by the ratio of the standard deviation over the mean of the observed variable X:

$$q_X = \frac{\sigma_X}{\mu_X} \tag{2}$$

called variation coefficient or relative dispersion. Moreover, some authors make use of the scatter parameter T, defined upon a measure of 10% and 90% percentiles of the observed variable X:

$$T_X = 1 : \left(\frac{X_{10\%}}{X_{90\%}} \right) ; \qquad \sigma_X \approx 0.39 \ln\left(\frac{1}{T_X} \right) \tag{3}$$

We assume that the standard deviation of logN distribution, $\sigma_{\ln N}$ is constant. Using of basic relations between associated normal and log-normal distributions [11] leads to:

$$\sigma_{\ln S} = \frac{\sigma_{\ln N}}{b} ; \qquad q_S = \frac{\sigma_S}{\mu_S} = \sqrt{e^{\sigma_{\ln S}^2} - 1} \tag{4}$$

so that the relative dispersion of the S distribution is expected to be constant too along the Basquin straight line:

$$\sigma_{\ln N} = const \quad \Rightarrow \quad \sigma_{\ln S} = const \quad \Rightarrow \quad q_S = const \tag{5}$$

As a result, from a graphical point of view, iso-survival probability lines are straight ones and parallel to the basic Basquin line, as highlighted in figure 2, right side.

The framework described above, equations (1,5), is now considered as a reference playground in order to discuss about the effectiveness of fatigue tests. Please note that we will not comment on failure probability as a result of a stress-strength interference method, but only focus on the identification of the strength parameters. With the preferential point of view that looks for the load level S distribution at a given reference lifetime N_{ref}. Moreover, under the constraint of a very limited number of available specimens: if it is common to test dozens of material specimens in order to provide a reliable material fatigue characterization, it may be difficult to obtain even only half a dozen of chassis system parts to be tested. As a consequence, it is allowed to look at the population kernel only, i.e. to describe its central portion by the two basic parameters μ and σ.

3. Fatigue tests: an overview

A fatigue test campaign is mainly composed by three ingredients, which drive its effectiveness. At first, the fatigue test protocol, i.e. the operating schedule used to practice the test, which is mainly responsible for the overall number of cycles required to complete a test campaign. Then, the estimation method applied to the test results in order to obtain a reliable value for the stress distribution, i.e. the mean fatigue strength, μ_S and its dispersion, q_S at a given lifetime. Please note that the fatigue test protocol and the fatigue strength estimation are, in principle, independent and may contribute separately to the effectiveness of a fatigue test campaign. At last, the number of tested specimens, n whose reduction may be allowed by the performance of the previous two items. In the sequel, despite their interaction, we try to discuss about these three ingredients separately.

3.1. Fatigue test specimen number

In this chapter we discuss about the compromise between the relevance of the statistic estimation and the required number of specimen, which is expected to be limited because of available testing time and cost. At first, this may be presented without any reference on the specific test procedure and the test estimation method applied.

No matter if the statistical distribution focuses on S or N, neither if it is supposed normal, log-normal or Weibull, it may be successfully described by two parameters, i.e. its mean value, μ and its scatter, expressed by the standard deviation, σ. Let a sample $X = [X_i]$ be provided, we may estimate the sample mean and the sample variance by basic unbiased estimators [11]:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i \;\; ; \;\; \hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \hat{\mu}\right)^2 \tag{6}$$

Let us introduce a confidence level, γ in order to identify relevant percentile estimations. A unilateral underestimation of the mean may be provided by a Student's t law with (n-1) degrees of freedom, whereas a unilateral overestimation of the variance may be provided by a χ^2 law with (n-1) degrees of freedom [11]:

$$\frac{\mu_{\min}}{\hat{\mu}} \approx \frac{t(1-\gamma; n-1)}{\sqrt{n}} \;\; ; \;\; \frac{\sigma^2_{\max}}{\hat{\sigma}^2} \approx \frac{\chi^2(\gamma; n-1)}{n-1} \tag{7}$$

Formulae (7) are plotted in figure 3 considering usual confidence levels of 90% (left side) and 75% (right side), respectively:

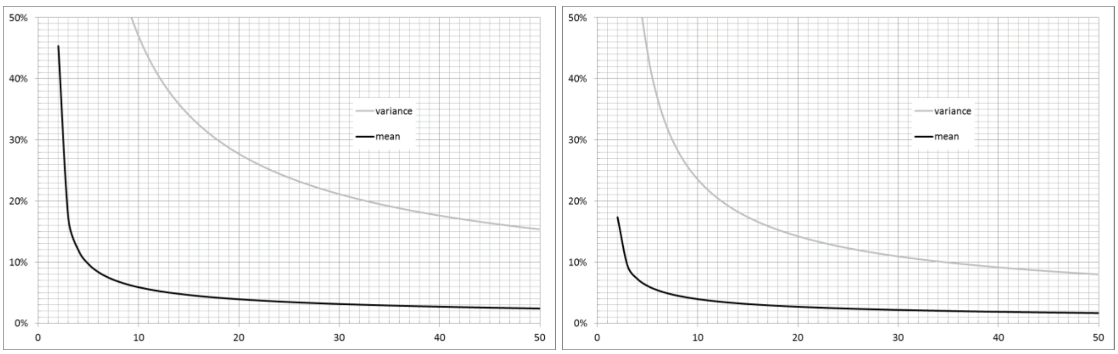

Fig. 3. Mean underestimation (black line) and variance overestimation (grey line) as a function of the specimen number n : 90% confidence level (left); 75% confidence level (right).

As expected, the larger the specimen number n is, the lower the underestimation of the mean is and the overestimation of the variance is, respectively. Moreover, with increasing n, a relevant statistics on the mean may be achieved earlier than on the variance. Nevertheless, it needs for more than n = 5 specimens in order to estimate the mean with less than 10% error and 90% confidence level, which is often considered as a standard for safety concerns. Adding one more specimen to the test campaign always offer the opportunity to improve the results relevance, i.e. to reduce the underestimation of the mean and the overestimation of the scatter, respectively. Conversely, this marginal gain is less useful as far as n increases. For instance, the mean estimation with 90% confidence level is improved by less than 1% for each additional specimen for n > 10.

The choice of a particular test procedure and/or a specific test estimation leads to improve the fatigue test reliability, as it is discussed in the sequel.

3.2. Fatigue test protocol

In this chapter we comment on the effectiveness of the fatigue test protocol, i.e. the operating schedule used to practice the test, in terms of the overall number of cycles required to complete a test campaign. Nevertheless, some preliminary remarks are also provided dealing with the estimation method applied to the test results in order to obtain a reliable value for the stress distribution, i.e. the mean fatigue strength, μ_S and its dispersion, q_S.

The easiest fatigue test procedure corresponds to perfect constant amplitude test, as proposed by the Stair Case schedule [12,13], see figure 4, left side, extensively applied for material specimen fatigue tests and usually completed by the estimation method proposed by Dixon & Mood [14]. Dealing with chassis parts, this procedure implies a risk of time loss if the load level is underestimated, leading to a lack of efficiency [10,15]. Moreover, the Dixon & Mood estimation may lead to an unreliable estimation of the scatter when applied to very few specimens [10,15], as is it often the case in an industrial context. Finally, at least half of the specimens are not tested until failure, leading to a lack of valuable information on the fatigue strength.

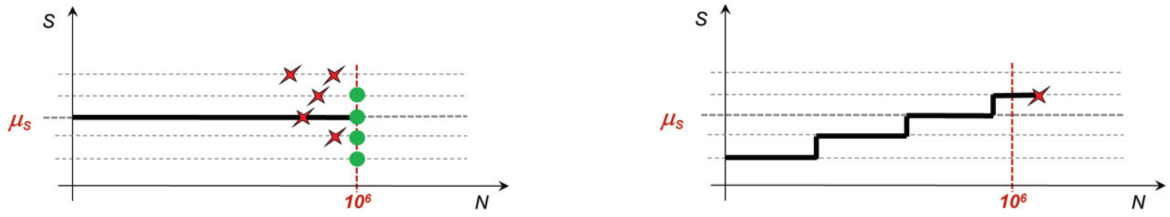

Fig. 4. Fatigue test protocols: Stair-Case (left); Locati (right).

In order to minimize all these shortcomings, the Locati fatigue test schedule [17] represents a valuable alternative, see figure 4, right side: by applying a step-wise increasing load amplitude, the specimen failure may be reached with a minimum loss of time [10,16]. Please note that it is an almost constant amplitude test, the deviation with respect to the mean fatigue strength, μ_S being of the order of magnitude of σ_S. The efficiency of this procedure is based on a wise choice of the schedule parameters, i.e. the starting point, the step length and the step height.

Obviously, it is not possible to achieve the best parameter choice by experiments. Conversely, numerical computations may be useful to deal with virtual test random generation [10,18]: based on the general framework presented before (§2), sample sets of fatigue test have been generated and submitted to analysis. At first to check the basic properties of the estimation (e.g. convergence and bias), then the sensibility to the test and model parameters (e.g. Basquin slope), and finally looking for the protocol parameter set minimizing the test duration (i.e. the overall cycle number involved). Figure 5 displays an example of virtual sample composed of 1000 random generated fatigue tests: looking at the mean fatigue strength, the theoretical value used for the random generation (red line) is well matched by the mean value of the 1000 estimations (black line), whereas some scatter appears, as highlighted by the percentiles values of the box-plot (left side).

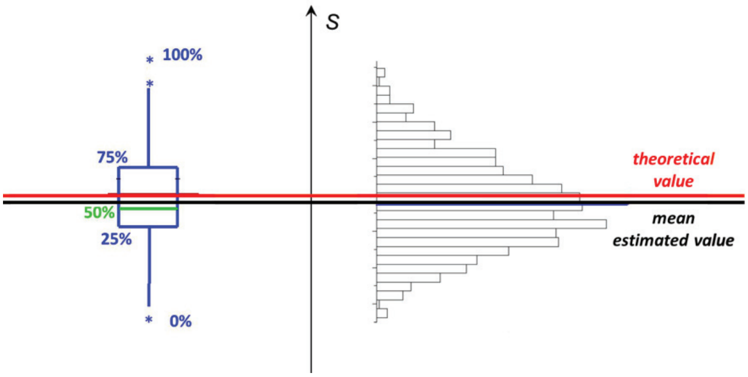

Fig. 5. Virtual fatigue test sample: mean strength population histogram (right); box-plot highlighting the population percentiles (left).

Applying this strategy, several points of interest may be addressed [10]:

- confirmation of the shortcomings of the StairCase protocol, as already announced above;
- check of the convergence without bias of the Locati protocol: the box-plot of the mean strength estimation is squeezed over the theoretical value for $n \rightarrow \infty$, as shown in figure 6, left side. Please note that the fatigue strength scatter estimation follows the same rule;
- measure of the estimation quality: the spread of the box-plot around the theoretical value describes how wrong the protocol results could be, i.e. the larger the box-plot, the coarser the estimation. Please note that because of the fatigue phenomenon is dispersive, it is absolutely possible for a test results to be away from the theoretical expected value;
- identify the best set of schedule parameters, i.e. the starting point, the step length and the step height, leading to the shortest fatigue test protocol. This work is achieved by extensive application of standard optimization techniques.

As a proof of such a comprehensive work, please note that since 2013 the practice of a new Locati protocol, instead of the old StairCase one, has been leading PSA Peugeot Citroën to save almost 10% testing time, for the same estimation reliability.

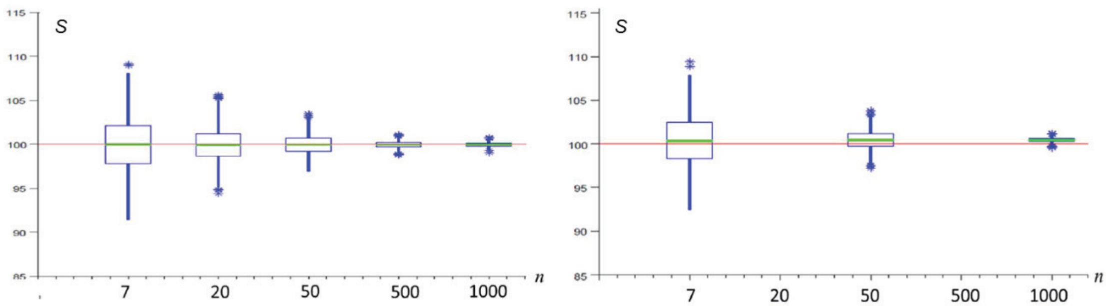

Fig. 6. Mean strength estimation from a Locati protocol as a function of the specimen number n : FEQ estimation (left); MLE estimation (right).

3.3. Fatigue test estimation method

In this chapter we will focus on different estimation methods available for, in principle, any fatigue test protocol. At first, a basic calculation of the fatigue strength of each specimen is achieved by applying the Basquin rule, equation (1) completed by the elementary linear damage rule of Palmgren [19] and Miner [20]. Considering a reference cycle number N_{ref} , the corresponding fatigue strength S_{eq} is calculated by:

$$S_{eq} = \left(\sum_i \frac{N_i}{N_{ref}} S_i^b \right)^{\frac{1}{b}} \tag{8}$$

where the index i applies in order to count for all the steps of the load history, e.g. in the case of a Locati protocol. Please note that this explicit calculation requires fixing the Basquin slope, according to the material of interest. Thus, the estimation of μ_S and σ_S of the overall test campaign may be provided by basic unbiased estimators, equation (6), for which confidence intervals are expressed by formulae (7). This approach, named FEQ, is explicit and robust indeed and leads to a reliable result even for a limited number of specimens, as shown by the fatigue strength mean estimation coming from a Locati protocol, figure 6, left side. Nevertheless, more sophisticated estimations may be provided by applying statistical tools like the Maximum Likelihood Estimation (MLE) and the Bayesian Approach (BAY).

The Maximum Likelihood Estimation (MLE) [21,22] is based on the following idea: a physical phenomenon (here, fatigue tests of chassis parts) is supposed to be described by a theoretical model (here, the framework presented in §2), which depends upon some parameters (here, the mean fatigue strength at a given N_{ref}, the associated scatter and possibly, the Basquin slope). So that, depending of the model parameters, it is possible to consider the probability to actually observe the collected test results. Thus, maximizing this probability with respect to the model parameters is expected to provide their reliable estimation.

The application of a MLE approach to a Locati protocol means to consider that each specimen of the fatigue test is independent from each other and always tested until failure. Thus, the likelihood function, L is the product of the probability density functions associated to each specimen j and reads [10,18]:

$$L(\mu_S, \sigma_s, b) = \prod_j \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{S_{eq}^j - \mu_S}{\sigma_S} \right)^2 \right] \tag{9}$$

where we call for the standard Gauss probability density function and S_{eq} comes from the previous equation (8). From a purely numerical point of view, please note that the maximization problem of the likelihood function is often transformed in the minimization of the opposite of the log-likelihood, which allows to make use of standard minimization library functions and to convert the product in equation (9) into a more handling sum.

A comparison between FEQ and MLE approaches to the same fatigue test protocol is provided thanks to the virtual test random generation technique, as shown in figure 6: the MLE mean strength estimation (right side) is at least as effective as the FEQ one (left side), the spread of the box-plots being slightly lower for the MLE ones.

Actually, the relevant advantage of the MLE approach is to consider the Basquin slope as a true parameter to be estimated, along with the mean strength and the strength scatter. As a consequence, the allowed better fit of the experimental data leads to minimize the scatter and improve its estimation.

As a proof of the proposed improvement, please note that since 2013 the practice of a new Locati protocol completed by MLE estimation, instead of the old StairCase protocol with FEQ estimation, has been leading PSA Peugeot Citroën to shorten the testing time of almost 15%, without any reduction of the results reliability.

4. Discussions & Conclusions

In this paper we dealt with fatigue tests applied to automotive passenger vehicles chassis parts in the high cycle fatigue range. In an industrial context, the key feature is to set an effective compromise between the number of specimens, i.e. the test duration and cost, and the statistical estimation relevance.

The optimization of the fatigue test protocol, calling for a tuned Locati schedule, and the improvement of the fatigue strength parameters estimation, calling for the Maximum Likelihood Estimation (MLE), is successfully applied at PSA Peugeot Citroën worldwide, in-house and by its suppliers, since 2013. The overall testing time reduction is of about 15% with respect to the old fatigue test protocol and estimation, and this without any degradation of the results reliability.

Moreover, using of the MLE approach gives the opportunity to another improvement: the application of the Bayesian inference technique [11,23,24]. Up to now in this paper, the fatigue strength parameters estimation was based on the actual fatigue test results only. In fact, in many cases there exists some complementary and useful information: general knowledge, expert experience, former fatigue test results on similar parts, etc. As soon as any data is made available in a quantitative way, i.e. under the form of a so-called “a priori” distribution of a parameter, $g(\vartheta)$ the Bayesian framework allows convoluting this information with the likelihood function of the same parameter, $L(X|\vartheta)$ (please read the fatigue test results, X as a function of the parameter, ϑ) in order to get a so-called “a posteriori” parameter distribution, $g(\vartheta|X)$ (please read the parameter, ϑ as a function of the fatigue test results, X). Whenever the “a priori” is at least as accurate as the likelihood, the “a posteriori” is expected to be more relevant, as shown in figure 7.

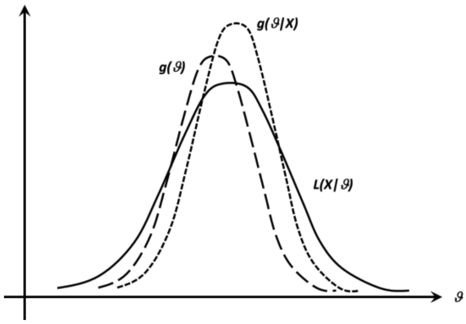

Fig. 7. Bayesian approach scheme applied to a parameter ϑ: the “a priori” distribution $g(\vartheta)$ is convoluted with the likelihood function of the same parameter, $L(X|\vartheta)$ in order to compute the “a posteriori” parameter distribution, $g(\vartheta|X)$.

From a mathematical point of view, the Bayesian approach reads:

$$g(\vartheta|X)=\frac{L(X|\vartheta)\,g(\vartheta)}{\int L(X|\vartheta)\,g(\vartheta)\,d\vartheta} \tag{10}$$

Of course, if no valuable “a priori” information is available, i.e. $g(\vartheta)$ is a uniform distribution, no improvement is expected and the “a posteriori” is nothing but the likelihood, $g(\vartheta|X) = L(X|\vartheta)$.

The application of the Bayesian inference technique is currently on-going at PSA Peugeot Citroën: thanks to the knowledge coming from hundreds of fatigue test campaigns over several decades of chassis system part design, a valuable “a priori” data on the mean fatigue strength, its scatter and also the Basquin slope is available indeed.

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